CONTROL ORIENTED TRANSIENT-DYNAMIC MODELING
AND SIMULATION OF METAL V-BELT CVT

by

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ABSTRACT

ROHAN VASUDEV BHATE. Control oriented transient-dynamic modeling and simulation of a metal V-Belt CVT. (Under the direction of DR. NILABH SRIVASTAVA)

With growing demand for environment friendly technologies, automobile manufacturers today are increasingly focusing on ‘Continuously Variable Transmissions’ (CVTs) as an alternative to conventional gearbox transmission; to achieve a balance between fuel economy and vehicle performance. By allowing for a continuous band of gear ratios between the driver shaft and driven shaft, a CVT permits the engine to operate for the most part in a region of high combustion efficiency resulting in lower emissions, and higher fuel economy. A change in clamping force acting on the variable-diameter pulley sheaves causes a variation in transmission ratio of the system due to change in belt pitch radius on the driver and driven pulley. To obtain a required variation in the transmission ratio, a control strategy which is capable of predicting the necessary clamping force under given operating conditions is desired. Understanding and capturing in sufficient detail, the dynamic interactions occurring in a CVT system during transient ratio-shifting states is a critical first step in developing such control strategies. The focus of this research is to develop transient dynamic models of metal V-belt CVT that facilitate the development of efficient and accurate control strategies. The challenge has been to develop a system-level simulation model which is relatively quick yet accurate enough to predict the power transmission behavior and inertial dynamics of a metal pushing V-belt CVT at transient state and steady state operating condition. This thesis report details a transient-dynamic model of a metal V-belt CVT that captures the various
transient dynamic interactions occurring in the system, including belt slip, belt inertial effects and influence of clamping force on the varying transmission ratio when the system is subjected to constant input torque on the driver pulley and constant load torque on the driven pulley. In the first part of this thesis a system-level model of a metal V-belt CVT under the constraint of constant driver pulley speed is introduced (to study the circumstance when the engine speed is restricted to an optimum fuel economy region). Although valuable insight regarding the CVT dynamic behavior can be obtained from this model it remains inadequate for the development of control algorithms because of the restriction on the driver-side pulley actuation force to maintain constant driver pulley speed, which amounts to creating a look-up table (showing required axial force at different driver pulley speeds). To overcome this shortcoming, the constraint of constant driver pulley speed was removed. The power transmission characteristics of a metal V-belt CVT system subject to constant driven pulley clamping force is investigated. Subsequently a comprehensive equation is derived which reveals an insightful relationship between transmission ratio of the system and the axial force on the movable pulley sheaves as other system parameters undergo variation. This relationship could serve as a powerful tool for the development of a more reliable/accurate CVT ratio shift controller. The range of pulley axial forces that allow the CVT system to meet required loading conditions (the operating range) is examined. The effect of CVT loading conditions on the transmission ratio profile is studied.
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NOMENCLATURE

$F_z$  pulley axial force

$F$  normal force between band pack and the element

$I$  pulley rotational inertia

$N$  normal force between the pulley and the element

$Q$  belt element compression

$r$  belt radius

$T$  band pack tension

$\nu$  parameter indicating difference between belt and pulley speed

$\alpha$  wrap angle

$\beta$  half-sheave angle of the pulley

$\beta_s$  half-sheave angle of the pulley in the sliding plane

$\theta$  element angular location on the pulley wrap

$\mu_a$  coefficient of friction between belt element and band pack

$\mu_b$  coefficient of friction between belt element and pulley

$\sigma_b$  linear mass density of band pack

$\sigma_e$  linear mass density of belt element

$\tau_{ld}$  driver input torque

$\tau_{in}$  driven load torque
$\phi$  slope angle of the belt element

$\psi$  sliding angle of the belt

$\omega$  angular speed of the pulley

$d$  distance between driver and driven pulley shaft

Superscript

$'$  driven side quantities
CHAPTER 1: INTRODUCTION

1.1 Need for Continuously Variable Transmission (CVT)

In an automobile, a gearbox provides a range of gear ratios between the engine and drive wheels, enabling the engine output to meet almost all load demands. Most commonly used types of automobile transmission are manual, automatic or semi-automatic transmissions. In conventional manual or automatic transmissions, power is transmitted from the engine to the drive wheels thorough a fixed set of gear combinations that vary the wheel speed and torque in relation to the engine speed and torque. A CVT enables a continuous band of gear ratios between the driver side and driven side of the transmission system. Thus a CVT enhances the fuel economy and acceleration performance of a vehicle by allowing the engine to operate at or near its best specific fuel consumption rate for variable driving scenarios. This is illustrated in Figure 1.

FIGURE 1 - Comparison of conventional gearbox and CVT
“In theory, the CVT has the ability to be the most fuel-efficient kind of transmission due to the infinite ability to optimize the ratio and operate the engine at its most efficient point” [1]. As the implementation of stringent regulations (Corporate Average Fuel Economy [CAFE] standards in the US) to reduce dependence on foreign oil and decrease tailpipe emissions come into effect, automotive engineers are increasingly looking at CVTs as a low cost and fuel efficient alternative to the conventional automatic gearbox. The major advantages of a CVT are higher engine efficiency, smooth and continuous acceleration, better fuel economy and relative ease of manufacturing (due to fewer parts). Since CVT is a friction limited drive, one major drawback is limited torque capacity, but the component parts have undergone appreciable development in recent years to be able to meet the requirement of automotive applications of up to 350Nm.
1.2 Metal V-Belt Continuously Variable Transmission

Currently three prevalent CVT technologies in use as automotive transmission systems are the belt CVT, the chain CVT and the toroidal CVT. Amongst all the CVT configurations available in the market, the belt and chain CVTs are most commonly used because of simplicity in design and low manufacturing cost. In this thesis the transmission system under consideration is a metal V-belt CVT.

Figure 2 [29] depicts a schematic of a metal V-belt CVT. The metal V-belt CVT is a friction limited drive; composed of two variable diameter pulleys kept a fixed distance apart and connected by a power transmitting metal V-belt. The application of an axial force to the movable pulley sheave causes belt displacement in the pulley groove causing the transmission ratio to vary continuously. The pulley clamping force and the CVT torque loading conditions influence the belt movement, which undergoes both radial and tangential motions.

FIGURE 2 - Schematic of metal V-belt CVT [29]
The V-belt consists of approximately 300 to 400 metal belt elements which are held in place by two sets of continuous, thin, flexible steel bands referred to as ‘band packs’. Figure 3 [2] depicts the structure of metal V-belt.

![Figure 3 - Structure of metal V-belt](image)

Torque is transmitted from the driver pulley (pulley on the engine side) to the driven pulley (pulley on the wheel side) via the ‘push-pull’ mechanism of the composite structured belt. ‘Pushing’ occurs when a belt element is pressed against another in front of it, due to the friction at the pulley-belt element interface, thus transmitting power from by means of the pushing action of the belt elements. Also, because of friction at the band pack-belt element interface, the band pack acts as like power transmitting flat belt. The ‘pulling’ action (i.e. the tension in the bands) also contributes to torque transmission. Research [19,24] has shown that the combined action of the tensile force in the band pack and the compressive force between the belt elements contributes to the torque transmission.
1.3 A literature review on dynamics and control of metal V-belt CVT

Continuously variable transmission is a promising powertrain technology which offers a continuum of gear ratios between fixed limits, improving the fuel economy and acceleration performance of a vehicle by allowing the engine to operate at or near its best specific fuel consumption rate for variable driving scenarios. A recent report [1] by the National Highway Traffic Safety Administration of the Department of Transportation states, “In theory, the CVT has the ability to be the most fuel-efficient kind of transmission due to the infinite ability to optimize the ratio and operate the engine at its most efficient point. However, this effectiveness is reduced by the significant internal losses from the high-pressure, high-flow-rate hydraulic pump, churning, friction loss, and the bearing losses required to generate the high forces needed for traction”. Thus, inspite of the indubitable potential advantages of a CVT system, they have not been fully realized in a mass produced vehicle. To achieve better performance it is necessary to understand the dynamic interactions occurring between various components of the CVT so that optimum control strategies could be developed which could predict required clamping force to attain desired transmission ratio and overcome the existing losses and enhance vehicle performance. In the last few decades, much interest had been devoted to the study of different aspects of CVTs such as configuration and design, vibration and noise analysis, stress and fatigue analysis of component parts, CVT mechanics and developing control strategies for a CVT system.

The focus of this research is to develop transient dynamic models of metal V-belt CVT that facilitate the development of efficient and accurate control strategies. The challenge has been to develop a system-level simulation model which is relatively quick
yet accurate enough to predict the power transmission behavior and inertial dynamics of a metal pushing V-belt CVT at transient state and steady state operating condition.

Dynamic Modelling of belt CVT –

Most of the models used to describe the behavior of the CVT systems, barring a few, are based on the principles of quasi-static equilibrium. Although these models are unable to accurately capture the transient-dynamic interactions occurring in a CVT system, analysis of the steady state behavior of CVT system lays the foundation for further development and has yielded valuable information on the torque transmission mechanism, belt force distribution and steady state operating regime.

Kobayashi et al. [19] investigated the torque transmission mechanism of a metal CVT belt. A simulation procedure for predicting the slip-limit torque of the belt based on the $\mu$-$v$ (friction coefficient versus sliding velocity) characteristic between the elements and pulley under steady state quasi-static equilibrium condition was developed. The effect of pulley clamping force on the slip ratio and slip-limit torque was studied and a method to determine suitable clamping force was deduced. The research reported on the redistribution of gaps between the belt elements. It was hypothesized that micro slip occurs between belt elements and the primary pulley when the elements are pushed together in the process of transmitting torque and that no slip would occur where no gaps existed between any of the belt elements.

Fujii, Kanehara and co-workers [20-23] through a number of experiments have studied the influence of tensile force in the band pack, compressive force in the belt elements and the pulley thrust on the transmitted torque at quasi-static steady state condition.
Srivastava et al. [24] developed a steady state model to study the operating regime of a metal V-belt CVT. The influence of loading conditions on the belt slip behavior and torque transmitting capacity as well as the axial force required to initiate torque transmission was investigated. The influence of CVT torque loading conditions and pulley clamping forces on belt slip was examined.

Asayama et al. [25] developed a model based on steady state quasi-static equilibrium analysis to study the torque transmitting mechanism of a metal V-belt CVT by theoretically analyzing forces on the belt elements and the band pack and the clamping force on the pulleys. The microslip behavior of the belt was modeled using elastic deformation of the belt elements distributions, from analysis of the results of experiments and computer simulations.

Carbone et al. [4] introduced a model to study the CVT transient dynamics during rapid speed ratio variations. Non-dimensional equations were defined to theoretically determine the dynamic response of the system. Later the influence of pulley deformation was studied by Carbone et al. [5] on the shifting mechanism of metal V-belt CVT. Pulley deformation was described by a sinusoidal function. The behavior of the CVT under slow and fast shifting maneuvers was studied. Pulley bending was modeled based on Sattler’s model. Sattler [6] developed a model to study the efficiency of V-belt and Chain CVTs. Pulley deformation was modeled using finite element analysis.

Srivastava et al. [7-9] developed a detailed transient dynamic model of a metal pushing V-belt CVT to investigate the dynamic interactions occurring between the element-band interface and element-pulley interface of a CVT system. The effect of pulley deformation and friction characteristic at the belt-pulley interface was modeled.
Inertial interactions between the belt and the pulley were also accounted for. The influences of band pack slip, friction characteristic and pulley deformation on the dynamic performance and also the torque capacity and the axial force requirements of the V-belt CVT system were studied. The authors concluded from the results of their simulations that the CVT, being a highly non-linear mechanism, is capable of exhibiting varied performance under different friction characteristics of the contact zone. Their work reports on the slip dynamics between the band pack and belt element and reveals that slip between the two components leads to the formation of inactive arcs (i.e. region of the belt in contact with the pulley that does not aid in torque transmission) which leads to losses in torque transmission. The model developed by the authors, however, was valid for only one cycle of the belt element as it traveled from the inlet to the exit of the driver and driven pulley, and the belt dynamics in the free section were not modeled. Srivastava et al. [10] proposed that the CVT being a nonlinear mechanism needs a specific set of operating conditions, in order to successfully meet the load requirements. In order to find this set of operating conditions, the authors used genetic algorithms (GA) and highlighted its efficiency in capturing this set by comparing it with results generated from the Design of Experiments (DoE).

Akehurst et al [34,35,36] in a series of papers investigated the power transmission efficiency of CVTs. The loss mechanisms that occur due to slip at the belt element-band interface and the belt element-pulley sheave interface and losses due to the pulley sheave deformation were investigated. Experimental measurement of these torque losses was conducted. Theoretical models based on the investigation of the torque loss mechanisms
were proposed. It was shown that the proposed models agree well with previously reported trends of belt force.

**CVT Control**

An accurate and fast control methodology of transmission shifting dynamics will better meet the goals of enhanced fuel economy and reduced tail-pipe emissions by maximizing torque transmissibility and minimizing belt slip by improving CVT performance. The objectives of maximum torque transmissibility/appropriate drivability and the reduction of fuel consumption, two partially opposite features, makes the development of an optimum CVT control strategy a difficult task. The main aspect of control of a CVT system is to achieve a desired gear ratio profile by applying the required pulley clamping force. An accurate and comprehensive relationship between the axial force, operating conditions and the rate of change of transmission ratio is crucial for developing an optimum controls strategy. A clamping force only as high as necessary to minimize belt slip at given loading conditions will lead to enormous improvement in belt life and less power lost to the hydraulic actuating mechanism [11].

For optimum engine operation the engine should be operated as close as possible to the optimum operating line (OOL). As shown in Figure 4, the OOL for minimum fuel consumption is a line connecting the points at which the torque-speed combination achieves optimal fuel efficiency.
Liu et al. [12] and Pfiffner et al. [13] reported a survey of control strategies of a CVT system. The authors report three shift control strategies, viz., “single track” strategy, “speed envelope” strategy and “off the beaten track” strategy. The “speed envelope strategy” is so called because the operating area of the system is formed by two curves on the engine speed vs. the vehicle speed graph (Figure 5(a)) [13]. By choosing a low desired engine speed improvements in the fuel economy of the vehicle are obtained. The “single track approach” involves maintaining the engine operation along the peak efficiency curve generated during steady state operations (Figure 5(b)) [13]. The engine torque is brought to the quasi-static peak efficiency curve as fast as possible and the gear ratio is brought to the value which corresponds to the final operating point. “Off the beaten track strategy” is a variation of “single track strategy” in which two different transient trajectories are used, namely a performance mode or an economy mode (Figure 5(c)) [13].
Laan et al. [14] reported the basic principles and structure of model-based variator control and the results when applied to a CVT equipped vehicle. Later Laan et al. [27] give insight into loss during torque transfer due to adhesive wear at the belt-pulley contact. The authors report that occurrence of macro-slip is acceptable until a certain extent which allows the development of control strategies based on belt slip and permit lower clamping forces.

Bonsen et al. [16] developed and tested a slip controlled CVT in a production vehicle. In order to improve belt life and belt wear and tear, the authors developed a method to measure and control belt slip. A robust PID controller with optimal load
disturbance response is used. The efficiency of the CVT system is maximized by a regulating the clamping forces and maintaining slip to an amount where traction coefficient is maximum.

Simmons et al. [26] validated theoretical models for ratio shifting of CVT with experimental results. Based on the experimental results the authors use the model developed by Carbone et al. [5] to develop a more complete variator model considering belt slip, ratio-shifting dynamics and hydraulic actuation dynamics. Slip control technique is applied to reduce clamping forces which results in increase in the CVT efficiency.

Pfiffner et al. [28] developed a control oriented CVT model taking into account belt mass. The authors develop a low order ODE model which takes into account the belt mass, which is suitable for control applications. However the deformation of pulley sheaves was ignored and the authors assumed that no slip occurs between the belt and pulley.

Adachi et al. [15] modeled the CVT system as a linear first-order system and employed feedback control to ensure stable controller performance against deviation of parameters and time delay. To improve output response a feed-forward controller was included.

Zhou et al. [17] developed a mathematical model of metal V-belt CVT along with a vehicle dynamics simulation model. The authors implemented a speed ratio fuzzy controller with parameter variation. An integrated control consists of achieving minimum fuel consumption by operating the engine at optimal operation point by simultaneously regulating the throttle valve opening and the CVT speed ratio.
Kim et al. [18] reported an integrated control considering powertrain loss and response lag of the CVT. To reduce the effect of CVT ratio response to the operating conditions the authors proposed compensation algorithms. The experimental results indicated that the optimal engine operation performed better than optimal torque compensation with regard to engine operation while giving nearly the same acceleration response.
1.4 Research objectives & accomplishments

Since the inherent advantage of using a CVT system is the improved fuel economy and higher engine combustion efficiency, extensive research opportunities exist in the area of maximizing the fuel economy of the vehicle by developing accurate ratio-shift controllers, minimizing slip losses and expanding/characterizing torque capacity of the CVT. An explicit simulation model of CVT dynamics will facilitate more accurate and robust control algorithms. A dynamic model of CVT can be developed by experimental analysis/identification techniques or by theoretical system analysis. The drawback of identifying various dynamic phenomenon by experimental analysis is that such a model will be limited to the specific CVT unit on which the experiment was conducted thus requiring a new analysis every time a different CVT system is studied. In comparison system-level physics based model of the CVT can be used to study different systems of the same configuration.

The goals of the current research are to implement control algorithms to commercially available automobile metal V-belt CVT units and to demonstrate improved ratio-shift control, increased fuel economy and better torque transmissibility as compared to currently used CVT control strategies. The development of a sufficiently detailed and accurate dynamic model of CVT system is a critical first step towards achieving these goals. Thus, the objective of this thesis work is the development and analysis of a transient-dynamic model of a metal V-belt CVT that accurately captures the interactions occurring between the various components of the system.

Figure 6 illustrates a general configuration and loading conditions of a V-belt CVT. Most reported CVT control algorithms assume negligible power loss during torque
transmission from driver to driven pulley and thus fail to accurately capture the dynamics accurately. In this thesis expression for friction torques ($\tau_f, \tau_f'$) has been developed to include the dependencies of axial force ($F_z$), the relative speed between belt and pulley ($\dot{v}$) and the belt radius ($r$). Further, time-dependent relations for transmission ratio change ($\dot{G}$) and the change in the relative velocity between the belt and pulley ($\dot{v}$) were developed. These equations which are developed using the various parameters of the CVT system enable the study of the ability of the system to meet load torque demand on the driven pulley under different operating conditions (i.e. operating regime). Consequently a system-level simulation model relatively quick and accurate to predict the power transmission behavior of the CVT and the change in transmission ratio was developed in MATLAB. It will further exploited to develop fast and robust control strategies at a later stage.

**Equations of motion**

\[
I_{in} \ddot{\omega} = \tau_{in}(m_{a/f}, \omega, t) - \tau_f(\mu, F_z, r, t) \\
I_{ld} \dot{\omega}' = -\tau_{ld}(t) + \tau_f'(\mu, F_z, r', \dot{r}', t) \\
\dot{G} \propto f(F_z, F_z', \dot{\omega}, \dot{r}', r, r', t, \mu)
\]

**FIGURE 6 – System level model for CVT ratio control**
In Chapter 3 a system level metal V-belt CVT model constrained to a constant driver pulley speed is developed to study the circumstance when the engine speed is restricted to an optimum fuel economy region. Chapter 4 covers a V-belt CVT model in which the pulley axial thrust behavior can be studied with respect to change in the transmission ratio, belt slip and the pulley angular speed.

The work being reported in this thesis has been published at various SAE and ASME conferences [30,31,32].
A metal V-belt CVT system consists of two variable diameter pulleys kept a fixed distance apart (refer Figure 7 [9]). Power is transmitted from the driver pulley (pulley on the engine-side) to the driven pulley (pulley on the wheel-side) by means of a power transmitting device, namely the metal V-belt.

The metal V-belt is composed of two multilayered steel band packs, which hold in place numerous trapezoidal metal belt elements. The torque loading conditions and the pulley actuation forces influence the nature of the belt movement on the pulley sheaves. The combined push (compressive force generated between the belt elements) and pull (due to tensile force in the bands) action aids in the transmission of torque from the driver pulley to the driven pulley.
The model presented in this paper is a further development of the work reported by Srivastava et al. [9]. The model development and analysis includes the following assumptions:

- Belt element and band pack is inextensible. Belt undergoes negligible deformation.
- Bending and torsional stiffness of the belt is neglected.
- The offset in the band pack and the belt element pitch radius is neglected (an approximation based on the observations of Srivastava et al. [9].
- Pulley deformation effects have been neglected.
- The belt element can undergo only compression and the band pack only tension.

![FIGURE 8 - Geometric description of belt drive](image)

The CVT model developed predicts the behavior of the system independent of the belt element position; it is not limited to one cycle of the belt element (i.e. the movement of a belt element from the entry to exit of a pulley). As illustrated in Figure 8, let the points $P$, $Q$, $R$ and $S$ represent the entry and exit points on the driver pulley and driven pulley respectively. Once the operating conditions at the inlet of the driver pulley are defined, the operating conditions at the driven pulley inlet are known due to the continuity of the V-belt and the dynamics in the free section of the belt. However since in
the proposed model, the belt dynamics in the free section of the belt have been ignored, to ensure continuity of the simulation when belt element exits the pulley, it is hypothesized that the magnitude of band tension and belt element compression at the inlet of the pulley are equal to those of the belt element exiting the preceding pulley at that instant. If the belt element exits at \( Q \) it is assumed that the band tension and belt element compression at \( P \) are equal to those at \( S \). This hypothesis is reasonable given that the free section of the V-belt does not contribute significantly to the torque transmission dynamics of the CVT system.

Figure 9 [9] shows the kinematic description of a belt segment engaged with the driver pulley. The plane ABE (refer Figure 9(a)) is the rotational plane of the belt. The plane ADC is the plane in which the belt slides as it moves radially and tangentially in the pulley groove. The sliding angle of the belt is denoted by \( \psi \). The local belt sliding velocity \( V_s \) has the components \( \dot{r} \) and \( r\omega_s \), where \( \omega_s \) is the local sliding angular velocity of the belt, defined later in this section. The pulley’s half opening angle (the wedge angle of the wedge-shaped space between the two pulley sheaves) is denoted as \( \beta \) and in the sliding plane as \( \beta_s \).

The kinematic description involves two coordinate systems (refer Figure 9(b)) \((n, \tau)\) attached to the belt and describes its path (\( \rho \) is the radius of curvature of the belt element) and \((e_r, e_\theta, e_z)\) which is a rotating coordinate system attached to the centre of the pulley O. The slope angle of the belt element is denoted by \( \varphi \). \( \theta \) and \( r \) denote the angular and radial co-ordinate of the belt respectively. Since the belt follows a non circular path as it traverses the pulley wrap, the centre of curvature of the belt path changes with time and is labeled C.
From the kinematic description of the belt the following relationships are obtained.

\[
\tan \beta_z = \tan \beta \cos \psi \quad (1)
\]

\[
\tan \psi = \frac{r \omega_s}{r} \quad (2)
\]

\[
\tan \phi \approx \phi \approx \frac{r}{r \dot{\theta}} \quad (3)
\]

Assuming that the belt always enters and exits the pulley tangentially, the wrap angles (as illustrated in Figure 8) can be found to be:

\[
\alpha = \pi - 2 \sin^{-1} \left( \frac{r' - r}{d} \right) \quad (4)
\]

\[
\alpha' = \pi + 2 \sin^{-1} \left( \frac{r' - r}{d} \right)
\]

Neglecting rate of change of slope angle (\( \phi \)) of the belt (since \( \phi \) undergoes small variations over the pulley wrap), one gets the following equations for the inextensibility of the belt element (as derived in [7,8,9]),
\dot{\theta} + r \dot{\theta} = 0 \tag{5}

Under the assumption of negligible flexural effects, small \( \phi \), and constant total belt length, the following constraint relationship could be readily obtained \([7,8,9]\), which couples the driven dynamics to the driver dynamics of the CVT system

\dot{\alpha} + \dot{\alpha}' = 0 \tag{6}

It is to be noted that although \( \dot{\theta} \) dynamics have been explicitly neglected during model development, the variation in \( \dot{\theta} \) is still accounted for by the following approximate dynamic relationship,

\dot{\theta} = v \omega \tag{7}

Where; \( v \) is the ratio of belt element to pulley angular velocity. Note for successful forward torque transmission \( v > 0 \). The local angular sliding velocity of the belt is \( \omega_s = \dot{\theta} - \omega \)

Figure 10 \([32]\) depicts the forces acting on the driver band pack and the driver belt element. Summing the forces in the normal and tangential directions yields the system of equations that describe the dynamic interactions between the band pack and belt element \([7,8,9]\).

\dot{T} + \mu_a |\dot{F}| \text{sgn}(r_2 - r_1) = 0 \tag{8}

\begin{align*}
T \dot{\theta} - \dot{F} &= \sigma_b (r^2 \dot{\theta}^3 + \dot{r}^2 \dot{\theta}) \tag{9} \\
\dot{Q} + 2\dot{N} \{ \sin \beta \sin \phi - \mu_b \cos \beta_s \sin(\phi + \psi) + \mu_a |\dot{F}| \text{sgn}(r_2 - r_1) \} &= 0 \tag{10} \\
-\dot{Q} \dot{\theta} - 2\dot{N} \{ \sin \beta \cos \phi - \mu_b \cos \beta_s \cos(\phi + \psi) \} + \dot{F} &= \sigma_c (r^2 \dot{\theta}^3 + \dot{r}^2 \dot{\theta}) \tag{11}
\end{align*}
(a) Free body diagram of driver band pack

(b) Free body diagram of driver belt element

FIGURE 10 – Free body diagram of the band and belt element [32]
Since the offset in radius between the belt element and the band pack has been neglected for simplicity (refer to [9] for detailed radial variations), the band pitch radius is assumed to be the same as the belt element pitch radius. Hence, there exists negligible relative velocity between the band pack and the belt element [9], or in other words, the band and belt element are assumed to move together. In the absence of relative velocity between the band pack and the belt element, a “signum” function \( \text{sgn} \) based on radii has been introduced in order to comply with the band tensile force trends well-mentioned in literature (Asayama et al. [20], Fujii et al. [25]).

Figure 11(a) depicts the forces exerted by belt element on the pulley sheave. Neglecting the dynamics due to sheave translational inertial effects, summing the forces acting on the movable pulley sheave in the axial direction and under the assumption of uniform pressure distribution between the belt element and pulley results in the following equation [9],

\[
F_z = \frac{Na}{\theta} (\cos \beta + \mu_b \sin \beta_s)
\]  

(12)

Figure 11(b) illustrates the torque acting on the driver pulley. Summing moments acting on the driver pulley, and assuming uniform pressure distribution yields the following torque-based equation for pulley speed dynamics [9],

\[
I \dot{\omega} = \tau_{in} + 2\mu_b r \alpha \cos \beta_s \sin \psi \frac{N}{\theta}
\]  

(13)

As mentioned before, parameter \( \nu \) has been introduced in the model to account for the slip between the belt and the pulley. Substituting equation (7) in equation (5) and rearranging,

\[
\dot{\nu} \omega + r \nu \dot{\omega} + r \nu \omega = 0
\]  

(14)
(a) Forces of belt element on the driver pulley sheave

(b) Torque acting on driver pulley sheave

FIGURE 11 – Forces and torque acting on driver pulley sheave [32]
The earlier work [7,8,9,10] was further developed and using equations (6) and (9), the following comprehensive correlation was developed,

\[ (T - Q) - 2 \frac{dN}{d\theta} \lambda = (\sigma_b + \sigma_e)(r^2 \dot{\theta}^2 + \dot{r}^2) \]  

(15)

where \( \lambda = \{ \sin \beta \cos \phi - \mu_b \cos \beta \cos(\phi + \psi) \} \)

Equation (15) is crucial for the further development of a more reliable/accurate CVT ratio-shift controller since the relation of axial force \((F_z)\) to transmission ratio \((r'/r)\) under given operating/loading \((\tau_{in}, \tau_{id})\) conditions of the CVT is revealed. Moreover, closer inspection indicates that these equations well accommodate the inertial coupling effects between the belt and pulley. A detailed expression for \( \dot{r} \) is presented and analyzed in the subsequent sections for different CVT operating conditions. A significant advantage of equation (15) lies in the fact that it can be also used to explore and gain insight into the operating regime of the CVT. The operating regime of CVT is defined by the complete range of possible combinations of the operating conditions \((i.e. F_z', \tau_{id}, \tau_{in})\) for which successful transmission of torque from driver pulley to driven pulley can be expected.

The dynamic simulation model was developed in MATLAB using the differential equations that describe the CVT system. The simulation tracks the belt element as it cyclically traverses both the driver and driven pulley wrap and records the different, changing dynamic performance parameters. The initial operating conditions to start the simulation are band pretension and element precompression at pulley inlet, and the input and load torque condition at the pulleys. The computer simulation solves for \((r, u, \omega, T, Q, F_z, \dot{r})\) at every time step. The numerical simulation is computed by stepping though a small, fixed time interval and calculating the integral of the derivatives.
Consequent sections of this thesis address the analysis of the results of the dynamic simulation of CVT system during ratio change under constant driver pulley speed model and constant driven axial force model. Later a discussion of the operating regime of CVT is undertaken.
CHAPTER 3 – CONSTANT DRIVER PULLEY SPEED MODEL

3.1 Model Development

A CVT system is subject to an input torque from the engine and a load torque from the wheels. Torque transmission occurs from driver pulley to driven pulley due to the push-pull action of the composite metal V-Belt. In this simulation-model the driver pulley is maintained at a constant speed \((\omega)\) 2500rpm and is subjected to a constant input torque \((\tau_{in})\) 200Nm whereas a constant resisting load torque \((\tau_{ld})\) 100Nm is applied to the driven pulley. The assumption of constant driver pulley speed is reasonable as it enables one to study the behavior of a CVT system under the conditions when the engine speed is restricted to an optimum fuel economy region.

The belt CVT model is developed on the MATLAB platform. The program uses the initial operating conditions data to calculate the time histories of pulley speeds, belt slip, velocities etc. that describe the dynamic interactions between the belt and the pulley. The initial operating conditions required for the transient dynamic CVT model are: belt element precompression and band pretension at driver and driven pulley inlets. Inertial dynamics in the free section of the belt have been ignored, however the constraint of constant belt length still enables coupling between the driver and driven pulley systems.

Figure 12 lists in table format, the numerical values of selected parameters used in the simulations.
Due to the constraint of constant driver pulley speed ($\dot{\omega} = 0$), equations (14, 13) reduce to:

$$\dot{\nu} = -\frac{\dot{r} \nu}{r}$$  \hspace{1cm} (16)

$$\dot{N} = -\frac{\tau \dot{\omega}}{2\mu r a \cos \beta \sin \psi}$$  \hspace{1cm} (17)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half Sheave Angle, $\beta$</td>
<td>$15^\circ$</td>
</tr>
<tr>
<td>Linear band pack density, $\sigma_b$</td>
<td>1.5kg/m</td>
</tr>
<tr>
<td>Linear element density, $\sigma_e$</td>
<td>2.0kg/m</td>
</tr>
<tr>
<td>Initial Transmission Ratio, $\nu/r$</td>
<td>0.0889/0.0508 (m)</td>
</tr>
<tr>
<td>Coefficient of friction between band pack and element, $\mu_\beta$</td>
<td>0.25</td>
</tr>
<tr>
<td>Coefficient of friction between band pack and element, $\mu_e$</td>
<td>0.25</td>
</tr>
<tr>
<td>Center distance, $d$</td>
<td>0.5m</td>
</tr>
</tbody>
</table>

Expanding equation (15), we get;

$$(T - Q) - \left(\frac{-\tau}{\mu_b r a \cos \beta \sin \psi}\right) (\sin \beta \cos \phi - \mu_b \cos \beta \cos (\phi + \psi))$$

$$= (\sigma_b + \sigma_e) \left((r \dot{\theta})^2 + \dot{r}^2\right)$$  \hspace{1cm} (18)

Equation (18) is a comprehensive relation between rate of change of belt radius ($\dot{r}$), belt slip parameter ($\nu$), pulley angular speed ($\omega$), pulley actuation force ($F_z$), the belt tension ($T$), element compression ($Q$). Inertial coupling between the belt and pulley has been accounted for. This equation will serve a powerful tool during controller development.
3.2 Simulation Results and Interpretation

**FIGURE 13 - Element angular position & wrap angle vs. time**

Figure 13 depicts the variation of the wrap angle on driver pulley ($\alpha$) and driven pulley ($\alpha'$) as well as the entry and exit position (belt element position) of the belt element from the driver pulley ($\theta$) and driven pulley ($\theta'$) sheaves. The increasing wrap angle on the driver pulley indicates increasing belt radius on driver pulley, consequently the belt element takes longer time to traverse the pulley wrap. This can be attributed to the enforced condition of constant driver pulley speed. Since the driver pulley tends to accelerate due to the influence of high input torque, to mitigate the acceleration effects due to input torque the belt radius on the driver pulley increases causing it to generate
higher friction torque. It is to be noted from Figure 13 that the CVT system enters the forced steady state condition around 23ms as the wrap angles on the driver and driven pulleys remain constant thereafter. Forced Steady State indicates that the belt radius has attained either maximum or minimum physical bounds and no further change in the geometric transmission ratio can occur. Any further change in the CVT transmission ratio can occur only when operating conditions change ($\tau_m$ or $\tau_d$). It is quite obvious the results from this model are not limited to one cycle of the belt element. When the belt element exits either one of the pulleys, the parameter values $[T, Q, \theta]$ are reset as explained earlier (refer Figure 8).

Figure 14 illustrates the time histories of the driver pulley and the driven pulley belt radius. Transmission ratio ($G$) is defined as the ratio of driven belt-pitch radius to the driver belt-pitch radius. Initially, the transmission ratio $G$ is greater than 1. Since a parameter ($\nu$), which relates how fast the belt is moving relative to the pulley has been included in the system equations; there are two different and unequal types of transmission ratio possible. One is the geometric ratio, which is ratio of the belt radius on either pulley (i.e. $r'/r$ ) and the other is the speed ratio, viz. the ratio of the angular speed of the driver pulley and driven pulley (i.e. $\omega'/\omega$).

It is to be observed from Figure 14 that there exists a time lag between the pulley motion and the belt motion. This can be ascribed to the wedging action of the belt into the pulley as well as due to the torque loading conditions.
FIGURE 14 – Belt radius vs. time

As the movable pulley sheave advances in the axial direction, the belt element moves in the radial and tangential directions. The belt element accelerates under the influence of radial and tangential forces while traversing the pulley wrap. However, due to the wedging action of the belt in the pulley groove, there exists slip between the belt and the pulley, which consequently fosters relative motion between them. It was observed that a change in the magnitude of input torque affects the lag. An increase in the input torque applied to the driver pulley resulted in decrease in the observed lag as shown in Figure 15.

Operating Conditions: $\tau_{in} = 200\text{Nm}$, $\tau_{id} = 100\text{Nm}$, $\omega_{\text{driver}} = 2500\text{rpm}$

CVT lag
FIGURE 15 - Variation in lag time with input torque

This phenomenon of delay in the response of CVT to the operating conditions has been reported during practical trials of CVT, as reported by Wicke et al. [33]. The wedging action of the belt element in the pulley groove affects the performance of a CVT significantly.

Figure 16 illustrates the variation of the belt slip parameter ($\nu$) with time. The colored regions indicated the CVT system operating state. The time history of slip parameter illustrates the variation in the belt angular speed in relation to the pulley angular speed. Value of slip parameter less than one implies that belt is moving slower than pulley.
It is to be noted that initially (during the lag phase) the slip parameter remains fairly constant on both the driver and the driven pulleys. The slip of the belt on the driven pulley increases as the driven belt pitch radius approaches the minimum limits, after which it stays constant during the forced steady state conditions of the CVT system. It was observed that while keeping the input torque constant, an increase in the load torque resulted in an increase in the magnitude of slip on the driven pulley (asserting the observations made by Kobayashi et al. [19], Srivastava et al. [24]). Decrease in the driven belt radius results in decreased frictional torque on the driven pulley, accordingly a decrease in the driven pulley speed is observed, which thereby indicates that CVT system
is able to meet the load only partially. Note; the value of slip parameter on the driver pulley is less than one, indicating that the belt moves slower than the pulley.

![Belt velocity vs. time](image)

**FIGURE 17 - Belt velocity vs. time**

Figure 17 depicts the time histories of the tangential belt velocity. The expression for belt velocity is $v = \dot{r} \hat{e}_r + r \dot{\theta} \hat{e}_\theta$. Since $r \cdot v \cdot \omega = r' \cdot v' \cdot \omega' \cdot k$; where $k$ is some constant, the ratio of the tangential belt speed must be nearly constant. From the figure is evident that the ratio remains constant for the duration of the simulation (negligible variation of 0.45 percent).

Figure 18 illustrates the variation in the driven pulley angular speed with time. In this simulation the driver pulley speed is maintained at 2500 rpm, only the driven pulley
speed varies. For the CVT system to meet the load torque continually, an increase in the driven pulley angular speed before forced steady state operating condition should be observed with the progress of time. It is to be noted from Figure 18 that the angular speed of the driven pulley decreases towards the end of transient state operation; hence the CVT system is able to meet the load requirements only partially.

FIGURE 18 - Driven pulley angular speed ($\omega'$) vs. time

The decrease in the driven pulley angular speed could be attributed to the decrease in the belt frictional torque on the driven pulley as the belt slips radially inwards. The minor overall increase in the driven pulley observed around 23ms occurs when the CVT system undergoes transition from the transient state to the steady state conditions. However, the
steady-state condition mentioned above is an enforced condition as the driven belt pitch radius still reaches the minimum limit. Any further change in the transmission ratio or angular speed beyond this point is possible only by varying the operating/loading conditions (i.e. by varying the $\tau_{ld}$, $\tau_{in}$, and $\omega$).

FIGURE 19 - Effect of belt-pulley inertial coupling on belt acceleration

Figure 19 depicts the time history of the belt acceleration as it traverses the pulley wrap. The trends emphasize the importance of detailed inertial coupling on the shift-ratio dynamics of the belt CVT. It is observed that when the belt pitch radius remains constant (i.e. at steady state), the belt-pulley inertial coupling does not affect the acceleration of the belt. However, during transmission ratio change, it is noted that there is a marked
difference in the trends of belt acceleration for a model accounting for inertial coupling and that without inertial coupling. This implies that detailed inertial effects and not just centripetal acceleration terms (as reported by Carbone et al. [5], Kobayashi et al. [19]) must be taken into consideration for accurately predicting CVT shift-ratio dynamics (especially in the transient regime).

Figure 20 illustrates the band tensile force time history. The tension in the V-belt also contributes to torque transmission. Fujii et al. [20], Kobayashi et al. [19], Asayama et al. [25], Srivastava et al. [24], have extensively reported the significance of band tensile force in determining the torque transmission behavior and load capacity of a metal V-belt CVT. Note the abrupt changes in the driven and driver belt tension time histories account for the transitions when the belt element leaves the pulley wraps. It is observed from Figure 13 that the belt element exits the driven pulley at 23ms. Henceforth, the values at the entry to driven pulley at the subsequent time step are updated as explained earlier (refer to Figure 8). From Figure 14 it is also observed that the transmission ratio changes to $G < 1$ at 15ms. Until this point, the belt tension on the driven pulley increases, while it decreases on the driver pulley (asserting the observations made by Kobayashi et al. [19], Asayama et al. [25]). Figure 14 indicates that the CVT system model enters forced steady state conditions at approximately 23ms. At this instant, the driven belt pitch radius is at the minimum limit and no further change in belt speed or radius is possible on either driver pulley or driven pulley unless the operating/loading conditions are varied. Under these conditions, the transmission ratio is maintained constant at $G < 1$. 
FIGURE 20 - Time history of band tension

The driven pulley belt tension force now decreases as the band traverses the pulley wrap at the lowest possible radius, while an increasing belt tension is observed on the driver pulley wrap. Again, the peaks in the driven belt tension time history are caused when the belt element exits the driven pulley. The parameter values for the element at the entry of the driven are reset to that at the exit of driver pulley for the subsequent time step and the simulation marches forward in time.
FIGURE 21 - Time history of belt element compression

Figure 21 depicts the time history of the belt-element compressive force. As compared to the pull of the band (tension in the band), the pushing action of the belt element (compression in belt element) contributes more to the torque transmission phenomenon in a metal V-belt CVT. The compressive force is generated as the steel elements push against each other. In this simulation, it is assumed that the belt element is never in tension and the band pack is never in compression. As the belt pitch radius increases on the driver pulley and as the belt speed gradually decreases (in order to abide by the constraint of constant driver pulley angular speed), elemental gap redistribution begins to happen (Kobayashi et al. [19], Srivastava et al. [24]), which consequently
causes a decrease in the belt compressive force profile over the driver pulley wrap. Certain regions of the pulley wrap where the force remains constant or the force is negligible, also known as idle arcs (refer Figure 21), were also observed to occur. These are regions of the pulley which do not actively participate in torque transmission. On the other hand, as the driven pulley radius decreases, the belt elements get tightly packed together, thereby generating a larger compressive force. As the elements stack/push upon each other tightly (due to lower radius and higher belt speed), the element compressive force increase from the inlet to the exit of the driven pulley wrap. The abrupt peaks in the time history of the compressive force again account for the transitions when the belt element leaves the pulley wraps (refer to Figure 13). It is thus observed that the trends of the tensile and compressive forces observed from the model are very much in consonance with the results reported in literature [5,7,19,21,25] thereby validating the use of this model to capture detailed inertial-coupling effects on CVT dynamics and facilitate the development of fast and more reliable CVT controllers.
CHAPTER 4 – CONSTANT DRIVEN PULLEY ACTUATION FORCE MODEL

4.1 Model development

A CVT system is subjected to an input torque from the engine and a load torque from the wheels. Torque transmission occurs from the driver to the driven pulley due to the push-pull action of the composite metal V-Belt. In this simulation the driven pulley actuation force is maintained at a constant speed \( (F_z') \) 30kN, driver pulley is subjected to an input torque \( (\tau_{in}) \) 200Nm and a resisting load torque \( (\tau_{ld}) \) 100Nm is applied to the driven pulley.

The belt CVT simulation was developed using MATLAB software. The program uses the initial operating conditions data to calculate the time histories of pulley speeds, belt slip, velocities etc. that describe the dynamic interactions between the belt and the pulley. The initial operating conditions required for the transient dynamic CVT model are: belt element precompression and band pretension at driver and driven pulley inlets. The constraint of constant belt length still enables coupling between the driver and driven pulley systems. Figure 12 lists the numerical values of selected parameters used in the simulations.

Expanding equation (15), and substituting operating conditions;

\[
(T' - Q') = \left( \frac{2F_z'}{\alpha' (\cos \beta_s' + \mu_b \sin \beta_s')} \right) \left( \sin \beta' \cos \phi' - \mu_b \cos \beta_s' \cos (\phi' + \psi') \right) \\
= (\sigma_b + \sigma_c) \left( (r'\dot{\theta})^2 + r'^2 \right)
\]

(19)
4.2 Simulation Results and Analysis

Figure 22 depicts the variation of the wrap angle on driver pulley ($\alpha$) and driven pulley ($\alpha'$) as well as the entry and exit point of the belt element from the driver pulley ($\theta$) and driven pulley ($\theta'$) sheaves. Decreasing wrap angle ($\alpha$) on the driver indicates that the belt pitch radius is decreasing and consequently it takes the belt element lesser time to traverse the pulley wrap. The plot is also indicative of the entry and exit points of belt element on each pulley. It is evident that the simulation is not restricted to one cycle of the belt element (i.e. movement of the belt element from entry to exit of pulley). As the wrap angle increases on the driven pulley the time taken to traverse the pulley wrap also
increases. It is observed that the angular speed of the belt element traversing the driven pulley decreases, approaching the speed of the pulley because as the belt radius on driven pulley increases, the higher frictional torque prevents driven pulley acceleration. The converse is observed on the driver pulley.

Figure 23 shows the time histories of the belt radius observed on driver pulley and driven pulley and the consequent geometric radius ratio. After running a number of simulations it was noted that the CVT system requires a specific set of operating conditions to ensure a favorable torque transmission behavior. It has been observed that the transmission ratio profile remains unchanged for various other combinations of driver input torque and driven load torque. Note that the variation in belt radius is in consonance with the constraint of constant belt length. As belt element moves from inlet to exit of either driver pulley or driven pulley, it moves not only tangentially but also radially in the pulley groove. The lag in the response of the belt radius to the operating conditions observed in Figure 23 occurs due to the wedging action of the belt element in the pulley groove. As a result of this wedging action there exists a time lag between the pulley motion and belt element motion. Similar phenomenon of delay in the CVT response to operating conditions has been reported in experimental trials (Wicke et al. [33]).
Figure 24 depicts variation in belt speed and the pulley angular speed. It is noted that during the transient state operation, belt moves slower than the pulley on the driver-side. This can be attributed to change in belt radius on the driver pulley, which decreases rapidly as a consequence of which driver belt friction torque decreases leading to increase relative speed between the belt and pulley. This phenomenon is also evident from Figure 25 which indicates that in the transient condition the belt on driver pulley undergoes gross slip relative to the pulley.
During transient state the belt speed changes rapidly until CVT system enters the forced steady state condition, when the belt moves with a constant but increased speed on the driver pulley and slows down on the driven pulley. It can be observed that the angular speed of a belt element traversing the driver pulley increases during transient state and then remains constant during steady state. During transient state the belt speed is dependent on $\nu$ and $\omega$. Due to the high input torque on the driver pulley, the driver pulley angular speed increases. The high rate of change of pulley angular speed causes the belt to slip on the pulley surface and an increase in the belt speed to match the pulley speed is observed. During the steady state condition the pulley speed remains constant
and thus no change in the belt speed is observed. The rate of change of pulley speed ($\dot{\omega}$) of the driven pulley is dependent on the load torque ($\tau_{ld}$), the belt sliding angle ($\psi$) and the normal force ($N$) between the belt and pulley sheave. The load torque on the driven pulley remains constant for the duration of the simulation at 100Nm. The varying sliding angle ($\psi$) and the normal force between belt and pulley sheave ($N$) dictate the behavior of the pulley speed response. The driven pulley speed remains fairly constant initially during the lag of CVT response to operating conditions. During the transient state driven pulley speed drops by about 25% and remains steady during the forced steady state. The magnitude of pulley actuation force ($F_a$) directly affects the normal force between the belt and pulley sheave. Note that initially the driver pulley speed increases, reaches a peak and then decreases to settle at a constant value during steady state. As the belt pitch radius on the driver pulley decreases the friction torque decreases causing increased slip between the belt and the pulley. As the driver pulley is subject to a constant high input torque of 200Nm, it begins to accelerate and reaches a peak value. At this point since the sliding angle ($\psi$) which depends on ($r, v, \omega, \dot{r}$) changes direction the driver pulley speed begins to decrease due to change in $\dot{r}$ and the normal force between the belt and the pulley ($N$) increases. During forced steady state condition since $\dot{r} = 0$, there is no further change in the pulley speed.

Figure 25 depicts the relative variation between in the belt speed and pulley speed when the driven pulley is applied a constant actuation force. The relation for belt speed is; $\dot{\theta} = v \omega$, where $v$ is a parameter which accounts for the difference in belt speed and pulley speed. $v > 1$ implies that belt moves faster than the pulley.
At the beginning of simulation it was assumed on both the driver pulley and driven pulley, that $\psi \approx 1$. On the driver pulley the belt is unable to generate a large enough frictional torque due to the influence of decreasing belt radius to oppose the high input torque and the driver pulley accelerates and the magnitude of $\psi$ increases.

Note that the belt moves slower on the driver pulley wrap, which as clarified earlier can be attributed to the change in the belt radius. Minor variations in the values ($\psi$) occur due to rapid successive counter-acting variations (as shown in Figure 26) in the sliding angle ($\psi$) during the lag state of the CVT simulation. $\dot{\psi}$ varies rapidly at higher values of $\psi$. 

**FIGURE 25 - Belt slip parameter ($\psi$) vs. time**

![Graph depicting the belt slip parameter ($\psi$) vs. time.](image)
driven pulley axial force due to the change in the sliding angle of the belt ($\psi$) such that there is in effect no net change in the value of the parameter $\nu$.

**FIGURE 26 - Sliding Angle ($\psi$) vs. time**

Since $\dot{\psi}$ is dependent on $\dot{\omega}$ and $\dot{r}$, a similar variation is observed for the angular speed ($\omega$) time histories. On the driver pulley side of the CVT system, towards the end of the transient state as the belt pitch radius increases, due to increased friction torque a drop in the driven pulley speed is observed. At forced steady state, the values of $\nu \approx 1$.

Figure 27 depicts the pulley actuation force time history. For the duration of the simulation the CVT model was subject to conditions of high input torque on the driver
pulley and lower load torque on the driven pulley. The driven pulley actuating force was maintained constant.

The value of axial force obtained for the driver pulley is the force required at movable pulley sheave to maintain the belt radius at that instant. The pulley actuation force ($F_z$) is dependent on normal force ($N$) and the pulley sheave angle ($\beta$ and $\beta_s$). Belt slip on the driver pulley decreases with increase in the axial force. An increase in the normal force in between the belt elements and the driver pulley, results in an increase in the driver axial force and the tendency of the belt to move radially. As belt pitch radius in the driven pulley increases to maximum permissible limits, the CVT system model enters into a

![FIGURE 27 - Pulley axial force vs. time](image)
forced steady state, wherein the transmission ratio remains constant until there is a change in the operating conditions of the CVT system (i.e. input/load torque, axial force on driven pulley). During this state the driver pulley actuation force restricts further displacement of the pulley sheave in order to keep the V-belt at the minimum possible driver belt pitch radius and the system’s tendency to decrease the radius further. Thus, a positive force is expected to prevent any further displacement of the movable pulley sheave. During forced steady state pulley sheave angle in the sliding plane ($\beta_s$) becomes zero since rate of change of belt pitch radius ($\hat{r}$) reduces to zero, the driver pulley axial force is dependent only on the normal force ($N$) between the belt and pulley.
FIGURE 28 – Operating regime for constant $F_z'$ model

Figure 28 depicts a 3-D rendering of the bounds on the maximum and minimum allowable constant driven pulley axial thrust ($F_z'$) that permits successful torque transmission for duration of dynamic simulation under given operating conditions ($\tau_{in}, \tau_{id}$). The ‘operating regime of CVT’ is the range of possible combinations of the operating conditions (i.e., $F_z', \tau_{id}, \tau_{in}$) for which successful transmission of torque from driver pulley to driven pulley ensues. The case under study is the constant driven pulley axial force model. Driver pulley sheave experiences varying axial force during the
simulation. Under this operating condition, to transmit power to the driven pulley sufficient to meet the load torque the belt radius on the driven pulley increases as indicated in Figure 29. The range and nature of change in the geometric transmission ratio for a range of driven pulley axial force values at constant load and input torque is depicted in Figure 29.

FIGURE 29 – Transmission ratio vs. driven axial force

The figure illustrates the complete range of permissible driven pulley axial force values under given torque loading conditions for which torque is successfully transmitted from the driver pulley to driven pulley during the course of dynamic simulation. It was observed that although the overall trend of the transmission ratio profile remains the
same; variations in the lag time, slope of curve (during ratio shifting transient state) and the time to forced steady state occurred with change in the constant driven pulley axial force. The increase in the lag time with increase in the constant driven pulley actuation force can be attributed to the phenomenon of wedging on the belt in the pulley sheaves. The wedging action of the belt element in the pulley groove affects the performance of CVT significantly (as reported by Wicke et al. [33]). Increase in the driven axial force, causes the pulley sheaves to be pressed harder against the belt (i.e. the wedging of the belt in pulley sheave) and longer lag times result. As is indicated by the Figure 25 where, it is clearly observed that the belt slip parameter ($\nu$) values are close to 1 and also from Figure 23 where no change in the belt radius is observed ($\dot{\rho} \approx 0$) which implies that belt velocity in the radial direction negligible and thus the relative motion between the pulley sheave and belt is minimal. As the belt attains the radius limit on either pulley it enters a forced steady state. During this condition no further change in the belt radius ($\dot{\rho} = 0$) occurs, and the belt slip parameter ($\nu$) approaches 1 and remains there as long as no change in the torque loading condition occurs and the driven pulley force remains unchanged. The time to forced steady state in influenced by the rate at which the belt radius changes ($\dot{\rho}$) and the limits on the belt radius itself. A greater value of ($\dot{\rho}$) implies higher slope (i.e. the slope of curve in transient state region). The plot indicates that as the constant driven axial force increases the slope of the curve in the transient region also increases which implies that as $F_2'$ increases $\dot{\rho}$ also increases. Since the belt radius on the driven side increases to transmit greater torque, a higher driven pulley axial force results in a greater value of rate of change of belt radius.
Figure 30 depicts the nature of change in the geometric transmission ratio profile, when the CVT system is subject to conditions of constant load torque and constant input torque and driven pulley clamping force. With increase in load torque, the rate of change of transmission ratio during the transient state increases. Due to effect of increasing belt radius on driven pulley, as friction torque increases any increase in the load torque contributes to the retardation of the pulley speed. With reduced speed of pulley sheave magnitude of the belt slip relative to pulley sheave is much lesser at higher loads. Thus an increase in the rate of change of the belt radius is observed.
Figure 28 is a 3-dimensional representation of the maximum and minimum bounds of driven axial force with range of input and load torque conditions, for which torque transmission occurs during dynamic simulation of CVT system. Important points to be inferred from the figure are:

In case of low load torque acting on the driven pulley, the boundary for acceptable axial force values is initially narrow but widens as the load torque increases. At low load torque conditions to prevent belt slip a higher minimum axial force is required to initiate and maintain torque transmission. This condition is relaxed as the load torque increases upto a certain limit. Beyond this limit the minimum permissible driven axial force values increase again begin increasing as the load torque increases. This can be attributed to the fact that since belt radius increases on the driven pulley, in order to transmit greater torque, a large positive pulley axial force (which moves the pulley sheaves closer together) is required [30]. With increase in load torque acting on driven pulley, the value of minimum possible $F_z'$ increases.

At a constant load torque condition, with change in the input torque the maximum and minimum permissible driven axial force values undergo negligible variation. As load torque increases very little change occurs in the maximum permissible driven axial force.
CHAPTER 6: CONCLUSION & FUTURE WORK RECOMMENDATIONS

The automotive continuously variable transmission offers the benefits of higher fuel economy and better vehicle performance in comparison to the conventional transmission/gearbox. Increasingly stringent fuel consumption and emissions regulations are the major causes for renewed recent interest and development in CVT technology and its use as a practical automobile transmission. Variation in the forces acting on the movable pulley sheaves and the metal belt causes change in the transmission ratio between the engine and wheels. To accurately control the available wheel torque and speed, it is required to correctly compute the forces in the belt and on the pulley sheaves to enable the design of control algorithms which could maximize the performance of the CVT. The research work described in this thesis report provides simulation model of a metal belt CVT with the aim of understanding the various dynamic interactions occurring between the components parts.

In Chapter 1, the pros and cons of using a CVT as automobile transmission were discussed. An explanation of the working of a generic belt CVT was presented. Later the recent developments in the analysis of the dynamics and control of a metal V-belt CVT were highlighted. The lack of a sufficiently detailed system level model, which could capture the torque transmission characteristics and be further exploited to develop shift control algorithms, was the major motivation for this thesis. In Chapter 2 the theoretical modeling of metal V-belt CVT was described. Assumptions made for the development of
theoretical model are stated. The model is a further extension of the work done by Srivastava et al [8,9,10]. Functional forms of the frictional torque, dependent on system operating conditions were developed. A comprehensive equation that relates rate of change of transmission ratio to the pulley axial force, belt slip and other system parameters was developed. In Chapter 3 a CVT system model constrained to constant driver pulley speed was introduced. The constant driver pulley speed model was studied to better understand the behavior of CVT system, when the engine speed is restricted to a region of higher efficiency. The simulation tracked the dynamic interactions occurring between the belt and pulley sheaves as the belt element traversed the pulley wrap. The variation in the belt forces \((T, Q)\), belt radius \((r)\), the pulley speeds \((\omega)\) and the magnitude of relative motion between the belt and the pulley sheave \((\nu)\) was studied. The results obtained are in agreement with trends well mentioned in literature regarding CVT dynamics. It was observed that the belt radius increases on the driver pulley, to negate the acceleration of the pulley and maintain constant pulley speed. Physical phenomenon like the lag of the CVT system to the torque loading conditions and the forces in the belt has been observed and analyzed from the results of CVT model dynamic simulation. Chapter 4 describes CVT dynamic behavior subject to the conditions of constant driver pulley actuation force was studied Later a study of the operating regime of the CVT operating under the same conditions was studied (Chapter 5). Model with constant driven axial force is more suitable for controller development as it had a larger range of permissible driver pulley actuation force condition. It was also observed that not every combination of operating condition will meet the torque demand on the driven pulley. The Appendix
details the simulation procedure and code developed to study the behavior of CVT system analytically.

Further improvements in the model can be achieved by including the elastic deformation of the pulley sheaves, including an engine torque model, modeling slip between the band and the belt element. However, an experimental validation/trial of a metal V-belt CVT operating under prescribed conditions needs to be conducted. Verification of various system parameters will allow fine tuning of the model such that comparable measurements can be obtained from a computer simulation.

Work is underway on developing a computer controlled CVT test rig. The system under consideration consists of metal V-belt CVT sourced from a 2007 Nissan Maxima 3.5 SE. Currently preliminary studies are being conducted to understand the method of actuation/control of the pulley sheaves. The Nissan CVT is controlled by a transmission control module (TCM) via CAN signals from an ECU. The transmission control module receives the signals and determines the required line pressure and other system parameters and sends signals to the step motor and solenoids housed in a control valve body on the CVT unit. Figure 31 depicts the control valve body. Work is still in progress to understand the complete function and operation of the control valve body.

After the assembly and calibration of the CVT test-rig, comparison and validation of the CVT simulation will be conducted. Measurements of belt speed and position with respect to the pulley sheaves and the pulley axial force will give a greater understanding how the gear ratio change occurs. The theoretical model will be revised to better match experimental results. At a later stage, control strategies (fuel economy and torque
transmission maximization) will be implemented in the simulation model and tested on the test rig.

FIGURE 31 - CVT control valve body
BIBLIOGRAPHY


[31] Bhate, R., Srivastava, N., ”Dynamic Performance of a Metal V-Belt CVT during rapid shift-ratio conditions for control applications”, 2009 SAE World Congress and Exhibition, Paper No. 09PFL-1004, April 20-23, 2009, Detroit, MI, USA


APPENDIX A: Algorithm for CVT model simulation
APPENDIX B: MATLAB code of the transient-dynamic model of V-belt CVT

% #1 - INITIALIZE SYSTEM PARAMETERS
% CONSTANT DRIVEN PULLEY AXIAL FORCE PROGRAM

close all;clear;clc;
global tf tstep ifin Fz2 tin tld r1ini r2ini
global cd idr idn mua mub rhob rhoe beta

%MODIFY OPERATING CONDITIONS HERE
tf = 0.10 ;% FINAL TIME (s)
tstep = 1e-5 ;% INCREMENTAL TIME STEP (s) (FIXED 1E-5)
Fz2(1) = 30*1e3 ;% CONSTANT DRIVEN AXIAL FORCE (N)
tin(1) = 200 ;% INPUT TORQUE (Nm)
tld(1) = 100 ;% LOAD TORQUE (Nm)

tot_iter = tf/tstep;
time = 0:tstep:tf;
i = 1 ;% iteration count start from 1 not 0
ifin = (tf/tstep) + 1;

%DRIVEN CONSTANT AXIAL FORCE
Fz2(ifin) = 0;
for c = 2:ifin
    Fz2(c) = Fz2(1);
end

%DRIVER SIDE INPUT TORQUE
tin(ifin) = 0;
for c = 2:ifin
    tin(c) = tin(1);
end

%DRIVEN LOAD TORQUE
tld(ifin) = 0;
for c = 2:ifin
    tld(c) = tld(1);
end

%INITIAL DRIVER SIDE BELT PITCH RADIUS
r1ini = 0.0508; % (m) (viz. 2 inch)
%INITIAL DRIVEN SIDE BELT PITCH RADIUS
r2ini = 0.0889; % (m) (viz. 3.5 inches)

%OTHER CONSTANT PARAMETERS
cd = 0.5 ; % center distance, (m)
idr = 0.025 ; % 0.05 kg m^2; m=4kg, r=6.5 inch
idn = 0.025 ; % kg m^2;
mua = 0.25 ; % coefficient of friction between element and band pack
mub = 0.25 ; % coefficient of friction between belt element and pulley
rhob = 1.5 ; % kg/m
rhoe = 2 ; % kg/m
beta = pi/12 ; % radians; half sheave angle

%END OF PROGRAM #1 (init.m)
% #2 MAIN PROGRAM
% 1ST ITERATION

i = 1;

r1(i) = r1ini;
r2(i) = r2ini;

alpha1(i) = (pi) - (2*(asin((r2(i) - r1(i))/cd)));
alpha2(i) = (pi) + (2*(asin((r2(i) - r1(i))/cd)));

% INITIAL DRIVEN SIDE EQUATIONS
r2dot(i) = -0.025;
v2(i) = 0.98;
w2(i) = 750*pi/30; %initial driven pulley speed

theta2dot(i) = v2(i)*w2(i);
theta2(i) = 1e-3;

phi2 = atan(r2dot(i)/(r2(i)*v2(i)*w2(i)));
psi2(i) = myangle(r2dot(i),(r2(i)*w2(i)*(v2(i)-1)));
betas2 = atan(tan(beta)*cos(psi2(i)));

lambda21 = sin(phi2)*sin(beta) - (mub*cos(betas2)*sin(phi2+psi2(i)));
lambda22 = cos(phi2)*sin(beta) - (mub*cos(betas2)*cos(phi2+psi2(i)));

Q2(i) = 1500;
dN2(i) = Fz2(i)/(alpha2(i)*(cos(beta)+(mub*sin(betas2))));
T2(i) = Q2(i) + (2*lambda22*dN2(i))+(rhob+rhoe)*
(((r2(i)*v2(i)*w2(i))^2)+((r2dot(i))^2));

dF2(i) = rhoe*(((r2(i)*v2(i)*w2(i))^2) + (r2dot(i))^2)) +
(2*dN2(i)*lambda22) + Q2(i);
\[ dQ2(i) = (((-2*\text{dN2}(i))*\lambda21(i)) - (\mu2*\text{abs}(\text{dF2}(i))*\text{sign}(r2(i)-r1(i)))) \];

\[ dT2(i) = (\mu2*\text{abs}(\text{dF2}(i))*\text{sign}(r2(i)-r1(i))) \];

\[ \text{w2dot}(i) = (-tld(i) + (2*\mu2*r2(i)*\alpha2(i)*\cos(\beta2)*\sin(\psi2(i))*\text{dN2}(i)))/\text{idn}; \]

\[ \text{v2dot}(i) = -1*((\text{r2dot}(i)*\text{v2}(i)*\text{w2}(i))+(\text{r2}(i)*\text{v2}(i)*\text{w2dot}(i)))/(\text{r2}(i)*\text{w2}(i)); \]

% DRIVEN SIDE PARAMETER ARRAYS --
\[
\begin{align*}
\text{r2} & = [\text{r2}(i)]; \\
\text{r2dot} & = [\text{r2dot}(i)]; \\
\text{v2} & = [\text{v2}(i)]; \\
\text{v2dot} & = [\text{v2dot}(i)]; \\
\text{w2} & = [\text{w2}(i)]; \\
\text{w2dot} & = [\text{w2dot}(i)]; \\
\text{theta2} & = [\theta2(i)]; \\
\text{theta2dot} & = [\theta2dot(i)]; \\
\alpha2 & = [\alpha2(i)]; \\
\text{T2} & = [\text{T2}(i)]; \\
\text{Q2} & = [\text{Q2}(i)]; \\
\text{psi2} & = [\psi2(i)]; \\
\text{dN2} & = [\text{dN2}(i)]; \\
\text{dF2} & = [\text{dF2}(i)]; \\
\text{dQ2} & = [\text{dQ2}(i)]; \\
\text{dT2} & = [\text{dT2}(i)];
\end{align*}
\]

% INITIAL DRIVER SIDE EQUATIONS
\[
\begin{align*}
\text{r1dot}(i) & = (-\text{r2dot}(i)*(\alpha2(i)/\alpha1(i))); \\
\text{v1}(i) & = 0.98; \\
\text{w1}(i) & = (\text{r2}(i)*\text{w2}(i)/\text{r1}(i));
\end{align*}
\]
\theta_1(i) = v_1(i) \cdot w_1(i);
\theta_1(i) = 1e-3;

T_1(i) = (3/2) \cdot T_2(i);
Q_1(i) = (3/2) \cdot Q_2(i);

\phi_1 = \arctan\left(\frac{r_1dot(i)}{r_1(i) \cdot \theta_1dot}\right);
\psi_1(i) = \text{myangle}(r_1dot(i), r_1(i) \cdot w_1(i) \cdot (v_1(i)-1));
betas_1 = \arctan\left(\tan(\beta) \cdot \cos(\psi_1(i))\right);

\lambda_{11}(i) = \sin(\beta) \cdot \sin(\phi_1) - \mu_b \cdot \cos(betas_1) \cdot \sin(\phi_1 + \psi_1(i));
\lambda_{12}(i) = \sin(\beta) \cdot \cos(\phi_1) - \mu_b \cdot \cos(betas_1) \cdot \cos(\phi_1 + \psi_1(i));

d_{N_1}(i) = \frac{(T_1(i) - Q_1(i) - ((\rho_b + \rho_{e})(r_1(i) \cdot v_1(i) \cdot w_1(i))^2 + (r_1dot(i))^2))}{2 \cdot \lambda_{12}(i)};

F_{z_1}(i) = \frac{d_{N_1}(i) \cdot \alpha_1(i) \cdot (\cos(\beta) + (\mu_b \cdot \sin(betas_1)))}{2 \cdot \lambda_{11}(i)};

d_{F_1}(i) = \rho_{e} \cdot ((r_1(i) \cdot v_1(i) \cdot w_1(i))^2 + (r_1dot(i))^2))
+ (2 \cdot d_{N_1}(i) \cdot \lambda_{12}(i)) + Q_1(i);

d_{Q_1}(i) = \frac{(\mu_a \cdot \text{abs}(d_{F_1}(i) \cdot \text{sign}(r_2(i) - r_1(i)))) + (2 \cdot d_{N_1}(i) \cdot \lambda_{11}(i))}{r_1dot(i) - r_1(i)};

d_{T_1}(i) = \frac{-\mu_a \cdot \text{abs}(d_{F_1}(i) \cdot \text{sign}(r_2(i) - r_1(i)))}{r_1dot(i) - r_1(i)};

w_1dot(i) = \frac{(t_{in}(i) + (2 \cdot \mu_b \cdot r_1(i) \cdot \alpha_1(i) \cdot \cos(betas_1) \cdot \sin(\psi_1(i)) \cdot d_{N_1}(i)))}{r_1dot(i) - r_1(i)};

v_1(i) = \frac{-1 \cdot (((r_1dot(i) \cdot v_1(i) \cdot w_1(i)) + (r_1(i) \cdot v_1(i) \cdot w_1dot(i)))}{r_1(i) \cdot v_1(i) \cdot w_1dot(i))};

% DRIVER SIDE PARAMETER ARRAYS
r_1 = [r_1(i)];
r_1dot = [r_1dot(i)];
v_1 = [v_1(i)];
v_1dot = [v_1dot(i)];
\begin{verbatim}
wl = [wl(i)];
w1dot = [w1dot(i)];
theta1 = [theta1(i)];
theta1dot = [theta1dot(i)];
Fz1 = [Fz1(i)];
psi1 = [psi1(i)];
alpha1 = [alpha1(i)];
T1 = [T1(i)];
Q1 = [Q1(i)];
dN1 = [dN1(i)];
dF1 = [dF1(i)];
dQ1 = [dQ1(i)];
dT1 = [dT1(i)];

%initialising loop
for i = 2:ifin
clc;
itr_no = i
if w2(i-1) > 0 && w1(i-1) > 0 && isreal(r2dot(i-1)) && v1(i-1) > 0 && v2(i-1) > 0

%reassigning
r1x = r1(i-1);
r1dotx = r1dot(i-1);
v1x = v1(i-1);
v1dotx = v1dot(i-1);
w1x = w1(i-1);
w1dotx = w1dot(i-1);
theta1x = theta1(i-1);
theta1dotx = theta1dot(i-1);
\end{verbatim}
Q1x = Q1(i-1);
psi1x = psi1(i-1);
dN1x = dN1(i-1);
dF1x = dF1(i-1);
dQ1x = dQ1(i-1);
dT1x = dT1(i-1);

r2x = r2(i-1);
r2dotx = r2dot(i-1);
v2x = v2(i-1);
v2dotx = v2dot(i-1);
w2x = w2(i-1);
w2dotx = w2dot(i-1);
theta2x = theta2(i-1);
theta2dotx = theta2dot(i-1);
alpha2x = alpha2(i-1);
T2x = T2(i-1);
Q2x = Q2(i-1);
psi2x = psi2(i-1);
dN2x = dN2(i-1);
dF2x = dF2(i-1);
dQ2x = dQ2(i-1);
dT2x = dT2(i-1);

Fz2x = Fz2(i);
tldx = tld(i);
tinx = tin(i);

rmin = 0.0127; % 0.5inch
rmax = 0.1651; % 6.5inch

if r2x <= 1.1*rmin || r1x <= 1.1*rmin || r2x >= 0.9*rmax
|| r1x >= 0.9*rmax
[r1_a,r1dot_a,v1_a,v1dot_a,w1_a,w1dot_a,alph1_a,thetal_a,thetaldot_a,dN]
1_a,Fz1_a,dF1_a,dT1_a,dQ1_a,psi1_a,T1_a,Q1_a,r2_a,r2dot_a,v2_a,v2dot_a,w
2_a,w2dot_a,alpha2_a,theta2_a,theta2dot_a,psi2_a,dN2_a,dF2_a,dT2_a,dQ2_a
,T2_a,Q2_a] = steady(r1x,r1dotx,v1x,v1dotx,w1x,w1dotx,theta1x,theta1dotx,Fz1x,dT1x,dQ1
x,T1x,Q1x,r2x,r2dotx,v2x,v2dotx,w2x,w2dotx,theta2x,theta2dotx,Fz2x,
dT2x,dQ2x,T2x,Q2x,tstep,cd,idr,idn,mua,mub,rhob,rhoe,beta,tldx,tinx);

else

[r1_a,r1dot_a,v1_a,v1dot_a,w1_a,w1dot_a,theta1_a,theta1dot_a,dN
1_a,Fz1_a,dF1_a,dT1_a,dQ1_a,psi1_a,T1_a,Q1_a,r2_a,r2dot_a,v2_a,v2dot_a,w
2_a,w2dot_a,alpha2_a,theta2_a,theta2dot_a,psi2_a,dN2_a,dF2_a,dT2_a,dQ2_a
,T2_a,Q2_a] = transt(r1x,r1dotx,v1x,v1dotx,w1x,w1dotx,theta1x,theta1dotx,dT1x,dQ1x,T1x
,Q1x,r2x,r2dotx,v2x,v2dotx,w2x,w2dotx,theta2x,theta2dotx,dT2x,dQ2x,T2x,Q
2x,tstep,cd,idr,idn,mua,mub,rhob,rhoe,beta,Fz2x,tinx,tldx);

end

r1        = [r1 r1_a];
r1dot     = [r1dot r1dot_a];
v1        = [v1 v1_a];
v1dot     = [v1dot v1dot_a];
w1        = [w1 w1_a];
w1dot     = [w1dot w1dot_a];
alpha1    = [alpha1 alpha1_a];
theta1    = [theta1 theta1_a];
theta1dot  = [theta1dot theta1dot_a];
Fz1        = [Fz1 Fz1_a];
T1        = [T1 T1_a];
Q1        = [Q1 Q1_a];
psi1      = [psi1 psi1_a];
dN1        = [dN1 dN1_a];
dF1        = [dF1 dF1_a];
dT1        = [dT1 dT1_a];
dQ1        = [dQ1 dQ1_a];
r2 = [r2 r2_a];
r2dot = [r2dot r2dot_a];
v2 = [v2 v2_a];
v2dot = [v2dot v2dot_a];
w2 = [w2 w2_a];
w2dot = [w2dot w2dot_a];
alpha2 = [alpha2 alpha2_a];
theta2 = [theta2 theta2_a];
theta2dot = [theta2dot theta2dot_a];
T2 = [T2 T2_a];
Q2 = [Q2 Q2_a];
psi2 = [psi2 psi2_a];
dN2 = [dN2 dN2_a];
dF2 = [dF2 dF2_a];
dT2 = [dT2 dT2_a];
dQ2 = [dQ2 dQ2_a];

end
end

load train
sound(y,Fs)

% END OF PROGRAM #2 (main.m)
function f = myangle(X,Y)

% PROGRAM #3-Returns the angle in radians corresponding to a point (x,y)

if X < 0 && Y < 0  % 3rd quadrant
    f = pi + abs(atan(abs(Y/X)));
end
if X > 0 && Y > 0  % 1st quadrant
    f = abs(atan(abs(Y/X)));
end
if X < 0 && Y > 0  % 2nd quadrant
    f = pi - abs(atan(abs(Y/X)));
end
if X > 0 && Y < 0  % 4th quadrant
    f = abs((2*pi)-atan(abs(Y/X)));
end
if X == 0 && Y ~= 0
    if Y > 0
        f = pi/2;
    else
        f = 3*pi/2;
    end
end
if Y == 0 && X ~= 0
    if X > 0
        f = 0;
    else
        f = pi;
    end
end
if X == 0 && Y == 0
    f = pi/2;
end
% END OF PROGRAM #3 (myangle.m)
function [r1_a,r1dot_a,v1_a,v1dot_a,w1_a,w1dot_a,theta1_a,theta1dot_a,dN1_a,Fz1_a,dF1_a,dT1_a,dQ1_a,psi1_a,T1_a,Q1_a,r2_a,r2dot_a,v2_a,v2dot_a,w2_a,w2dot_a,theta2_a,theta2dot_a,psi2_a,dN2_a,dF2_a,dT2_a,dQ2_a,T2_a,Q2_a] = transt(r1x,r1dotx,v1x,v1dotx,w1x,w1dotx,theta1x,theta1dotx,dT1x,dQ1x,T1x,Q1x,r2x,r2dotx,v2x,v2dotx,w2x,w2dotx,theta2x,theta2dotx,dT2x,dQ2x,T2x,Q2x,tstep,cd,idr,idn,mua,mub,rhob,rhoe,beta,Fz2x,tinx,tldx)

% PROGRAM #4 - TRANSIENT STATE EQUATIONS

format long g;
display('Transient State')

% DRIVEN SIDE VALUES -->

r1_a = r1x+(tstep*r1dotx);
r2_a = r2x+(tstep*r2dotx);

alpha2_a = pi+(2*(asin((r2_a-r1_a)/cd)));
alpha1_a = pi-(2*(asin((r2_a-r1_a)/cd)));

w2_a = w2x+(tstep*w2dotx);
v2_a = v2x+(tstep*v2dotx);

theta2dot_a = (v2_a)*(w2_a);
theta2_a = theta2x+(tstep*theta2dotx);

T2_a = T2x+(tstep*dT2x*theta2dotx);
Q2_a = Q2x+(tstep*dQ2x*theta2dotx);

Fz2_a = Fz2x;
tld_a = tldx;
tin_a = tinx;
% function to calculate value of r2dot
x0 = r2dotx;
va = r2_a;
vb = v2_a;
vc = w2_a;
vd = T2_a;
ve = Q2_a;
vf = alpha2_a;
options = optimset('Display','off');
x =
fsolve('r2dot',x0,options,va,vb,vc,vd,ve,vf,beta,mub,rhob,rhoe,Fz2_a);
if isreal(x)
    if abs(x)<1e-7
        r2dot_a = 0;
    else
        r2dot_a = x;
    end
else
    error('Complex r2dot')
end

psi2_a = myangle(r2dot_a,r2_a*((v2_a)-1)*w2_a);
phi2 = atan((r2dot_a)/(r2_a*v2_a*w2_a));
betas2 = atan(tan(beta)*cos(psi2_a));

lambda21 = sin(beta)*sin(phi2)-mub*cos(betas2)*sin(phi2 + psi2_a);
lambda22 = sin(beta)*cos(phi2)-mub*cos(betas2)*cos(phi2 + psi2_a);

dN2_a = Fz2_a/(alpha2_a*(cos(beta)+(mub*sin(betas2))));

dF2_a = rhoe*((r2_a*v2_a*w2_a)^2)+((r2dot_a)^2)+(2*dN2_a*lambda22)+ Q2_a;

if r1_a==r2_a
    dQ2_a = (-2*dN2_a*lambda21 - mua*abs(dF2_a));
\[ dT2_a = \mu_a \cdot \text{abs}(dF2_a); \]

else
\[ dQ2_a = (-2 \cdot dN2_a \cdot \lambda21 - \mu_a \cdot \text{abs}(dF2_a) \cdot \text{sign}(r2_a - r1_a)); \]
\[ dT2_a = \mu_a \cdot \text{abs}(dF2_a) \cdot \text{sign}(r2_a - r1_a); \]
end

\[ w2dot_a = \frac{((-tld_a) + 2 \cdot \mu_b \cdot r2_a \cdot \alpha2_a \cdot \cos(\beta2_a) \cdot \sin(\psi2_a) \cdot dN2_a)}{idn}; \]

\[ v2dot_a = -1 \cdot \frac{((r2dot_a \cdot v2_a \cdot w2_a) + (r2_a \cdot v2_a \cdot w2dot_a))}{(r2_a \cdot w2_a)}; \]

% CURRENT DRIVER SIDE VALUES -->

\[ r1dot_a = (-r2dot_a \cdot \alpha2_a) / \alpha1_a; \]

\[ w1_a = w_{1x} + (\text{tstep} \cdot w_{1dotx}); \]
\[ v1_a = v_{1x} + (\text{tstep} \cdot v_{1dotx}); \]
\[ \theta1dot_a = (w_{1x}) \cdot (v_{1x}); \]
\[ \theta1_a = \theta_{1x} + (\text{tstep} \cdot \theta_{1dotx}); \]

\[ T1_a = T_{1x} + (\text{tstep} \cdot dT_{1x} \cdot \theta_{1dotx}); \]
\[ Q1_a = Q_{1x} + (\text{tstep} \cdot dQ_{1x} \cdot \theta_{1dotx}); \]

\[ \psi1_a = \text{myangle}(r1dot_a, r1_a \cdot ((v1_a - 1) \cdot w1_a)); \]
\[ \phi1 = \text{atan}((r1dot_a) / (r1_a \cdot v1_a \cdot w1_a)); \]
\[ \beta1 = \text{atan}((\tan(\beta) \cdot \cos(\psi1_a))); \]

\[ \lambda_{11} = \sin(\beta) \cdot \sin(\phi1) - \mu_b \cdot \cos(\beta1) \cdot \sin(\phi1 + \psi1_a); \]
\[ \lambda_{12} = \sin(\beta) \cdot \cos(\phi1) - \mu_b \cdot \cos(\beta1) \cdot \cos(\phi1 + \psi1_a); \]

\[ dN1_a = (((T1_a - Q1_a) - ((r_{ho} + r_{ho}) \cdot ((r_{1_a} \cdot v_{1_a} \cdot w_{1_a})^2 + (r_{1dot_a})^2))) \]
\[ / (2 \cdot \lambda_{12}); \]

\[ dF1_a = r_{ho} \cdot (((r_{1_a} \cdot v_{1_a} \cdot w_{1_a})^2) + ((r_{1dot_a})^2)) + \]
\[ (2 \cdot dN1_a \cdot \lambda_{12}) + Q1_a; \]
% If r1<r2, T1 should decrease with increasing theta, whereas T2 should
increase
if r1_a==r2_a
    dQ1_a = -1*(-2*dN1_a*lambda11 + mua*abs(dF1_a));
    dT1_a = -mua*abs(dF1_a);
else
    dQ1_a = (-2*dN1_a*lambda11 + mua*abs(dF1_a)*sign(r2_a-r1_a));
    dT1_a = -mua*abs(dF1_a)*sign(r2_a-r1_a);
end

w1dot_a = (tin_a + 2*mub*r1_a*alpha1_a*cos(betas1)*sin(psi1_a)*dN1_a)
          /idr;

v1dot_a = -1*((r1dot_a*v1_a*w1_a)+(r1_a*v1_a*w1dot_a))/(r1_a*w1_a);

Fz1_a = dN1_a*alpha1_a*(cos(beta)+(mub*sin(betas1)));

%to check whether the element is about to leave the pulley(s)

if theta1_a >= alpha1_a && theta2_a >= alpha2_a % both driver and driven
    element exiting
    T1_a_new = T2_a;
    T2_a_new = T1_a;
    Q1_a_new = Q2_a;
    Q2_a_new = Q1_a;
    theta2_a = 1e-3;
    theta1_a = 1e-3;
    T1_a = T1_a_new;
    T2_a = T2_a_new;
    Q1_a = Q1_a_new;
    Q2_a = Q2_a_new;
end
if theta1_a >= alpha1_a % only the driver exiting
    T1_a_new = T1_a + dT2_a*alpha2_a;
    Q1_a_new = Q1_a + dQ2_a*alpha2_a;
    theta1_a = 1e-3;
    T1_a = T1_a_new;
    Q1_a = Q1_a_new;
end

if theta2_a >= alpha2_a % only driven exiting
    T2_a_new = T2_a + dT1_a*alpha1_a;
    Q2_a_new = Q2_a + dQ1_a*alpha1_a;
    theta2_a = 1e-3;
    T2_a = T2_a_new;
    Q2_a = Q2_a_new;
end

% Lower saturation limits for tension and compression
Qmin = 10;
Tmin = 10;
if Q1_a < 0
    Q1_a = Qmin;
end
if T1_a < 0
    T1_a = Tmin;
end
if Q2_a < 0
    Q2_a = Qmin;
end
if T2_a < 0
    T2_a = Tmin;
End

% END OF PROGRAM #4 (transt.m)
function f = r2dot(x,va,vb,vc,vd,ve,vf,beta,mub,rhob,rhoe,Fz2_a)

% PROGRAM #5 - Function to find r2dot

f = (vd-ve) - 
(2*Fz2_a/(vf*(cos(beta)+(mub*sin(atan(tan(beta)))*cos(atan(va*(vb-
1)*vc/x))))))*(sin(beta)*cos(atan(x/(va*vb*vc)))) - (mub *
cos(atan(tan(beta)*cos(atan(va*(vb-1)*vc/x)))))*(cos((atan(va*(vb-
1)*vc/x)+atan(x/(va*vb*vc)))))-(rhob+rhoe)*((va*vb*vc)^2 + x^2);

% END OF PROGRAM #5 - (rdot.m)
function [r1_a,r1dot_a,v1_a,v1dot_a,w1_a,w1dot_a,alpha1_a,theta1_a, theta1dot_a,dN1_a,Fz1_a,dF1_a,dT1_a,psi1_a,T1_a,Q1_a,r2_a,r2dot_a, v2_a,v2dot_a,w2_a,w2dot_a,alpha2_a,theta2_a,theta2dot_a,psi2_a,dN2_a,dF2 _a,dT2_a,dQ2_a,T2_a,Q2_a] = steady(r1x,r1dotx,v1x,v1dotx,w1x,w1dotx,theta1x,theta1dotx,Fz1x,dT1x,dQ1 x,T1x,Q1x,r2x,r2dotx,v2x,v2dotx,w2x,w2dotx,theta2x,theta2dotx,Fz2x,dT2x, dQ2x,T2x,Q2x,tstep,cd,idr,idn,mua,mub,rhob,rhoe,beta,tldx,tinx)

% PROGRAM #5 - STEADY STATE EQUATIONS

format long g;
display('Forced Steady State')

tld_a = tldx;
tin_a = tinx;

rmax = 0.1651;
rmin = 0.0127;

if r1x <= 1.1*rmin
    r1_a = 1.1*rmin;
    r2_a = r2x;
end
if r1x >= 0.9*rmax
    r1_a = 0.9*rmax;
    r2_a = r2x;
end
if r2x <= 1.1*rmin
    r2_a = 1.1*rmin;
    r1_a = r1x;
end
if r2x >= 0.9*rmax
    r2_a = 0.9*rmax;
\[ r_{1\_a} = r_{1\_x}; \]
\[ \text{end} \]

% Driven side equations
\[
\alpha_{2\_a} = \pi + (2 \times (\text{asin}((r_{2\_a} - r_{1\_a})/cd))); \\
\alpha_{1\_a} = \pi - (2 \times (\text{asin}((r_{2\_a} - r_{1\_a})/cd))); \\
w_{2\_a} = w_{2\_x} + (tstep \times w_{2\_dotx}); \\
v_{2\_a} = v_{2\_x} + (tstep \times v_{2\_dotx}); \\
\theta_{2\_dot_a} = (v_{2\_a})*(w_{2\_a}); \\
\theta_{2\_a} = \theta_{2\_x} + (tstep \times \theta_{2\_dotx}); \\
w_{1\_a} = w_{1\_x} + (tstep \times w_{1\_dotx}); \\
v_{1\_a} = v_{1\_x} + (tstep \times v_{1\_dotx}); \\
\theta_{1\_dot_a} = (v_{1\_a})*(w_{1\_a}); \\
T_{2\_a} = T_{2\_x} + (tstep \times dT_{2\_x} \times \theta_{2\_dotx}); \\
Q_{2\_a} = Q_{2\_x} + (tstep \times dQ_{2\_x} \times \theta_{2\_dotx}); \\
F_{z2\_a} = F_{z2\_x}; \\
r_{2\dotx_a} = 0; \\
\% \, \text{ws2} = w_{2\_a} \times (v_{2\_a} - 1); \\
\psi_{2\_a} = \text{myangle}(r_{2\dotx_a}, r_{2\_a} \times (v_{2\_a} - 1) \times w_{2\_a}); \\
\phi_{2} = \text{atan}((r_{2\dotx_a})/(r_{2\_a} \times v_{2\_a} \times w_{2\_a})); \\
betas_{2} = \text{atan}(\text{tan}(\beta) \times \cos(\psi_{2\_a})); \\
\lambda_{21} = \sin(\beta) \times \sin(\phi_{2}) - \text{mub} \times \cos(betas_{2}) \times \sin(\phi_{2} + \psi_{2\_a}); \\
\lambda_{22} = \sin(\beta) \times \cos(\phi_{2}) - \text{mub} \times \cos(betas_{2}) \times \cos(\phi_{2} + \psi_{2\_a}); \\
\text{dN}_{2\_a} = F_{z2\_a} / (\alpha_{2\_a} \times (\cos(\beta) + (\text{mub} \times \sin(betas_{2})))); \\
\text{dF}_{2\_a} = \rho_{e} \times (((r_{2\_a} \times v_{2\_a} \times w_{2\_a})^2) + ((r_{2\dotx_a})^2)) + (2 \times \text{dN}_{2\_a} \times \lambda_{22}) + \text{Q}_{2\_a};
if r1_a==r2_a
    dQ2_a = (-2*dN2_a*lambda21 - mua*abs(dF2_a));
    dT2_a = mua*abs(dF2_a);
else
    dQ2_a = (-2*dN2_a*lambda21 - mua*abs(dF2_a)*sign(r2_a-r1_a));
    dT2_a = mua*abs(dF2_a)*sign(r2_a-r1_a);
end

if (abs(theta2dot_a-w2_a)<0.0025*w2_a)
    w2dot_a = 0;
    v2dot_a = 0;
else
    w2dot_a = ((-tld_a) + 2*mub*r2_a*alpha2_a*cos(betas2*sin(psi2_a) *dN2_a)/idn;
    v2dot_a= -1*((r2dot_a*v2_a*w2_a)+(r2_a*v2_a*w2dot_a)) /
                        (r2_a*w2_a));
end

%Driver side equations
% w1_a = w1x+(tstep*w1dotx);
% v1_a = v1x+(tstep*v1dotx);
% thetal1_a = (v1_a)*(w1_a);
% thetal1_a = thetal1x+(tstep*thetal1dx);

T1_a = T1x+(tstep*dT1x*thetal1dx);
Q1_a = Q1x+(tstep*dQ1x*thetal1dx);

% Fz1_a = Fz1x:

r1dot_a = 0;
% ws1 = w1_a*(v1_a -1):

psil1_a = myangle(r1dot_a,r1_a*((v1_a)-1)*w1_a);
phil1 = atan((r1dot_a)/(r1_a*v1_a*w1_a));
betas1 = atan(tan(beta)*cos(psi1_a));

lambda11 = sin(beta)*sin(phi1)-mub*cos(betas1)*sin(phi1 + psi1_a);
lambda12 = sin(beta)*cos(phi1)-mub*cos(betas1)*cos(phi1 + psi1_a);

dN1_a = ((T1_a-Q1_a)-((rhob+rhoe)*((r1_a*v1_a*w1_a)^2+(r1dot_a)^2)))
      /(2*lambda12);

dF1_a = rhoe*(((r1_a*v1_a*w1_a)^2)+((r1dot_a)^2)+(2*dN1_a*lambda12)
      + Q1_a;

% If r1<r2, T1 should decrease with increasing theta, whereas T2 should increase
if r1_a==r2_a
    dQ1_a = (-2*dN1_a*lambda11 + mua*abs(dF1_a));%
    dT1_a = -mua*abs(dF1_a);
else
    dQ1_a = (-2*dN1_a*lambda11 + mua*abs(dF1_a)*sign(r2_a-r1_a));
    dT1_a = -mua*abs(dF1_a)*sign(r2_a-r1_a);
end

if (abs(thetadot_a-w1_a)<0.0025*w1_a)
    w1dot_a = 0;
    v1dot_a = 0;
else
    w1dot_a = (tin_a*mub*r1_a*alpha1_a*cos(betas1)*sin(psi1_a)
            *dN1_a)/idr;
    v1dot_a= -1*(((r1dot_a*v1_a*w1_a)+(r1_a*v1_a*w1dot_a))
            /(r1_a*w1_a));
end

Fz1_a = dN1_a*alpha1_a*(cos(beta)+(mub*sin(betas1)));

%to check whether the element is about to leave the pulley(s)
if theta1_a >= alpha1_a && theta2_a >= alpha2_a % both driver and driven
  element exiting
    T1_a_new = T2_a;
    T2_a_new = T1_a;
    Q1_a_new = Q2_a;
    Q2_a_new = Q1_a;
    theta2_a = 0.001;
    theta1_a = 0.001;
    T1_a = T1_a_new;
    T2_a = T2_a_new;
    Q1_a = Q1_a_new;
    Q2_a = Q2_a_new;
end

if theta1_a >= alpha1_a % only the driver exiting
  T1_a_new = T1_a + dT2_a*alpha2_a;
  Q1_a_new = Q1_a + dQ2_a*alpha2_a;
  theta1_a = 0.001;
  T1_a = T1_a_new;
  Q1_a = Q1_a_new;
end

if theta2_a >= alpha2_a % only driven exiting
  T2_a_new = T2_a + dT1_a*alpha1_a;
  Q2_a_new = Q2_a + dQ1_a*alpha1_a;
  theta2_a = 0.001;
  T2_a = T2_a_new;
  Q2_a = Q2_a_new;
end

% Lower saturation limits for tension and compression
Qmin = 10;
Tmin = 10;

if Q1_a < 0
  Q1_a = Qmin;
end
if T1_a < 0
    T1_a = Tmin;
end
if Q2_a < 0
    Q2_a = Qmin;
end
if T2_a < 0
    T2_a = Tmin;
End
%END OF PROGRAM #5 (steady.m)
plotit.m

% PROGRAM #6 – PLOTTING THE SIMULATION RESULTS

% xm = 3500; % value of xm depends on the total successful iterations

figure(1) % INPUT TORQUE & LOAD TORQUE (vs) TIME (tin and tld)
plot(time(1:xm),tin(1:xm),'r',time(1:xm),tld(1:xm),'LineWidth',2)
legend('Input Torque','Load Torque')

figure(2) % BELT ELEMENT ANGULAR POSITION (theta)
plot(time(1:xm),thet1(1:xm),time(1:xm),alpha1(1:xm),'r',time(1:xm),theta2(1:xm),'-',time(1:xm),alpha2(1:xm),':','LineWidth',2)
legend('	heta_1','\alpha_1','\theta_2','\alpha_2')

figure(3) % BELT PITCH RADIUS
plot(time(1:xm),r1(1:xm)*100/2.54,'r',time(1:xm),r2(1:xm)*100/2.54,':')
legend('Driver','Driven')

figure(4) % BELT SLIP PARAMATER (v)
plot(time(1:xm),v1(1:xm),'r',time(1:xm),v2(1:xm),':')
legend('Driver','Driven')

figure(5) % PULLEY ANGULAR SPEED (w)
plot(time(1:xm),w1(1:xm)*30/pi,'r',time(1:xm),w2(1:xm)*30/pi,':')
legend('Driver','Driven')

figure(6) % PULLEY AXIAL FORCE (Fz)
plot(time(1:xm),Fz1(1:xm),'r',time(1:xm),Fz2(1:xm),':')
legend('Driver','Driven')

figure(7) % BELT TENSION PLOT (T)
plot(time(1:xm),T1(1:xm),'r',time(1:xm),T2(1:xm),':')
legend('Driver','Driven')
figure(8) % BELT ELEMENT COMPRESSION PLOT (Q)
plot(time(1:xm),Q1(1:xm),'r',time(1:xm),Q2(1:xm),'r:')
legend('Driver','Driven')

figure(9) % RATE OF CHANGE OF BELT RADIUS PLOT (rdot)
plot(time(1:xm),r1dot(1:xm),'r',time(1:xm),r2dot(1:xm),'r:')
legend('Driver','Driven')

figure(10) % EFFECT OF INERTIAL COUPLING
a_m = (v1.*w1).*((r1.*v1.*w1).^2 + (r1dot.^2)).^(0.5);
% a_m = r1dot.^2./r1 + r1.*(v1.*w1).^2;
a_other = r1.*(v1.*w1).^2;
a_m2 = (v2.*w2).*((r2.*v2.*w2).^2 + (r2dot.^2)).^(0.5);
% a_m2 = r2dot.^2./r2 + r2.*(v2.*w2).^2;
a_other2 = r2.*(v2.*w2).^2;

plot(time(1:xm),a_m(1:xm),'r',time(1:xm),a_m2(1:xm),'b',time(1:xm),a_other(1:xm),'g',time(1:xm),a_other2(1:xm),'m')
legend('Driver with inertial coupling','Driven with inertial coupling','Driver w/o inertial coupling','Driven w/o inertial coupling')

figure(11) % GEOMETRIC RATIO PLOT (r2/r1)
tr = (r2./r1);
plot(time(1:xm),tr(1:xm),'r')

figure(12) % SPEED RATIO PLOT (w2/w1)
sr = (w2(1:xm)./w1(1:xm));
plot(time(1:xm),sr(1:xm))

figure(13) % PULLEY & BELT ANGULAR SPEED
plot(time(1:xm),w1(1:xm)*30/pi,'r',time(1:xm),w2(1:xm)*30/pi,'b',time(1:xm),v1(1:xm).*w1(1:xm)*30/pi,'r:',time(1:xm),v2(1:xm).*w2(1:xm)*30/pi,'b :','Linewidth',2)
legend('Driver pulley \omega_1', 'Driven pulley \omega_2', 'Driver belt \omega', 'Driven belt \omega')

% END OF PROGRAM #6 (plotit.m)